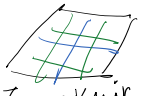
Session1

lunedì 26 luglio 2021 14:59

- Azuma.pdf (6) Inequality for supermartingales.
- BiparameterPT.pdf
- Brelot.pdf (7)
- CarlesonCounterexampleWick.pdf (2) $H^2(\mathbb{D}\times\mathbb{D}) \ni f(z,w)$: CARLESON CONDITION IS NOT SUFFICIENT
- CarlesonMeasuresHardyWick.pdf (1) CHARACTERIZATION OF CARLESON MEASURES FOR THE HARDY SPACE
- Choquet.pdf (7)
- ChristDecomposition.pdf (3)
- DyadicPTAhlfors.pdf
- MarcinkiewiczBiMax.pdf (9)
- MWinequality.pdf (5)
- Sawyer.pdf (8)
- Tao_Hausdorff dimension.pdf
- Wolff.pdf (4)

$\mathbb{D} = \{|z|<1\}$



X metric space

$f(z)$ HOLOMORPHIC IN $|z|<1$ S.T. $\lim_{r\to 1^-} \sup_{r<1} \int_{-\pi}^{\pi} |f(re^{it})|^2 \, dt < +\infty$ $= \|f\|^2_{H^2}$

$\mu \geq 0$ on $|z|<1$;

$$\boxed{\int_{-\pi}^{\pi} |f|^2 \, d\mu \leq C(\mu) \cdot \|f\|^2_{H^2}:}$$

Characterization Carleson Thm

AT THE END OF THE DAY

- $\mathbb{R}^3$; $p=2$; $\alpha=1$: $(-\Delta)V^\mu = \mu$:

$$V^\mu = I_1(I_1\mu) \qquad \mu(S(I)) \leq |I|$$

$$\boxed{\underset{\text{SCALAR}}{I_1\mu} \approx \underset{\text{VECTOR}}{\nabla V^\mu}: \quad \mathcal{E}(\mu) = \int \|\nabla V^\mu\|^2 \, d\mathrm{vol} = \int |I_1\mu|^2 \, d\mathrm{vol}}$$

- $\mathbb{R}^d$; $1<p<\infty$; $0<\alpha<d$

$0<\alpha\leq d$, $\alpha\in\mathbb{N}$

$$\boxed{\|f\|^p_{W^{\alpha,p}} = \sum_{|\beta|\leq\alpha} \|D^\beta f\|^p_{L^p} \iff f = G_{\alpha,p}\, g \quad \text{AND} \quad \|f\|_{W^{\alpha,p}} \overset{\text{Thm}}{\approx} \|g\|_{L^p}}$$

$\alpha\in\mathbb{N}$

What about μ? $\quad g = [G_{a,p}\mu]^{p'-1}$ See later on

$$f = G_{a,p}[G_{a,p}\mu]^{p'-1}$$

$$\|f\|^2_{H^2} \simeq |f(0)|^2 + \int_{|z|<1} \underbrace{|f'(z)|^2}_{|\nabla f(z)|^2} \cdot \underbrace{(1-|z|^2)}\, dx\, dy$$

$|z|<1$

weighted Sobolev norm in $|z|<1$ restricted to hol. functions

warm.

$$\|f\|^2_{D\text{ irrichlet}} \simeq |f(0)|^2 + \int_{|z|<1} |f'(z)|^2\, dx\, dy$$

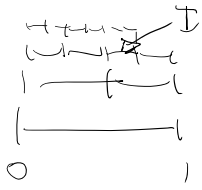
1 DERIVATIVE IN $|z|<1$

$$\| $$

$$\simeq |\hat{f}(n)|^2 (n+1) \quad : \quad \tfrac{1}{2} \text{ DERIVATIVE IN } |z|=1$$

$a = 1/2$

(4,5) $\quad \sum |a_n| \geq \left(\sum |a_n|^p\right)^{1/p} \geq \sup_n |a_n|$ VERY STRICT



I: DYADIC INTERVALS

$\mu \geq 0$ on $[0,1]$; $0<s<1$; $1 \leq p < \infty$

$$\int_0^1 \Big[\underbrace{\sum_{I \ni x} \frac{\mu(I)}{|I|^s}}_{\ell^2}\Big]^p dx \lesssim \boxed{\int_0^1 \sum_{I \ni x} \frac{\mu(I)^p}{|I|^{ps}}\, dx} \lesssim \int_0^1 \Big(\underbrace{\sup_{I \ni x} \frac{\mu(I)}{|I|^s}}_{(?)\ \ell^{\infty}}\Big)^p dx$$

RIESZ-BESSEL KERNEL

Wolff 1984

(4) combinatorial; dyadic probabilistic in spirit

MW 1974

(5) good-λ

(7) CHOQUET CAPACITABILITY THEOREM.

we define $\mathrm{cap}(E)$ $\forall E \subseteq X = \mathbb{R}^n$ or not.

(sort of) outer measure.

- $\mathrm{Cap}(E) = \inf_{U \supseteq E \text{ open}} \mathrm{Cap}(U) \quad \forall E$

$\mathrm{Cap}(E) \overset{?}{=} \sup_{K \subseteq E \text{ compact}} \mathrm{Cap}(K)$

Thm. TRUE IF E IS BOREL (SUSLIN SET) $\boxed{F_\sigma \quad G_\delta}$

(8) ERIC SAWYER 1985:

Weighted Hardy inequality: conditions on μ, ν:

$$\int_0^{+\infty} \Big[\underbrace{\int_0^y f(x)\,dx}_{F(y)}\Big]^p d\mu(x) \le C(\mu,\nu) \cdot \int_0^{+\infty} f(x)^p \, d\nu(x)$$

$F'(x)$

TALENTI 1960..

$$\int_0^{+\infty}\int_0^{+\infty} \Big[\int_0^{x_1}\int_0^{x_2} f(x_1,x_2)\,dx_1\,dx_2\Big]^{\overset{2}{\overset{\shortparallel}{p}}} d\mu(y_1,y_2) \le$$

$$\le \int_0^{+\infty}\int_0^{+\infty} f(x_1,x_2)^2 \, \cancel{d\lambda(x_1,x_2)} \, dx_1\,dx_2$$

$\Longleftrightarrow$??. GEOM. - COMBINATORIAL.

$$\sum \Big(\sum_{\substack{n_1 \in m_1 \\ n_2 \in m_2}} f(n_1,n_2)\Big)^2 \mu(m_1,m_2) \le \sum f(n_1,n_2)^2$$

$\hookrightarrow$ short research paper ???!. $\boxed{\$\$\$\$}$

$\$\$\$$ OPEN

(9) BIPARAMETER MAXIMAL FUNCTIONS.

(9) BIPARAMETER MAXIMAL FUNCTIONS.

ONE-PARAMATER: $f : [0,1] \longrightarrow \mathbb{R}^+$

MAX. FUNCT. $\pm$ dyadic. $\quad Mf(x) = \sup_{I \ni x} \frac{1}{|I|} \int_I f \, dm$ (HARDY LITTLEWOOD MAX. FUNCT.)

<u>Thm 1</u> $\int |Mf|^2 \, dm_\mu \leq C_1 \cdot \int f^2 \, dm_\mu$. W.L.

<u>Thm. 2</u> $M_\mu f(x) = \sup_{I \ni x} \frac{1}{\mu(I)} \int_I f \, d\mu$: <u>semi</u>.

<u>BIP.</u> $f : [0,1]^2 \longrightarrow \mathbb{R}_+$

$Mf(x) = \sup \frac{1}{|R|} \int_R f \, dm \; : R = I \times J$

<u>Thm 1</u> ≥ 0's MARC.-ZYG.-JESSEN

$$\iint (Mf)^2 \, dm \leq C_1 \cdot \int f^2 \, dm$$

<u>Thm 2</u>. is FALSE $\longrightarrow$ CAIROLI? EXERCISE WITH HINT.

Question: $\mu \geq 0$ on $[0,1]^2$: Thm. 2 holds!

$$d\mu(x_1, x_2) = d\mu_1(x_1) \cdot d\mu_2(x_2) \; : \underline{\text{TRUE}}.$$

(10) Riesz Rep. Thm. $[C_b(X)]^* \cong \text{Meas}_{\text{bnd}}(X)$

Continuous
Bounded

$[C_0(X)]^*_+ \cong$ Radon measures(X)

vanishing @ ∞

RUDIN
REAL & COMPLEX ANALYSIS
chapt. III?